

Discrete-Time Periodicity

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A discrete-time signal $x[n]$ is periodic if

$$x[n + N_0] = x[n] \text{ for all } n. \quad N_0 \text{ is positive.}$$

The smallest value of N_0 is the fundamental period. For a two-sided cosine signal,

$$x[n] = \cos(\omega_0 n) \text{ where } \omega_0 = 2\pi \frac{f_0}{f_s} = 2\pi \frac{N}{L}$$

where N and L are relatively prime integers, and f_0 is the continuous-time frequency and f_s is the sampling rate.

$$\begin{aligned} x[n + N_0] &= \cos\left(2\pi \frac{N}{L} (n + N_0)\right) \\ &= \cos\left(2\pi \frac{N}{L} n + 2\pi \frac{N}{L} N_0\right) \\ &= \cos\left(2\pi \frac{N}{L} n\right) = x[n] \end{aligned}$$

if $2\pi \frac{N}{L} N_0$ is an integer multiple of 2π , i.e.

if $\frac{N}{L} N_0$ is an integer. The smallest value of N_0 is $N_0 = L$. Fundamental period is L .

There are N continuous-time periods in the fundamental discrete-time period.

Spring 2014